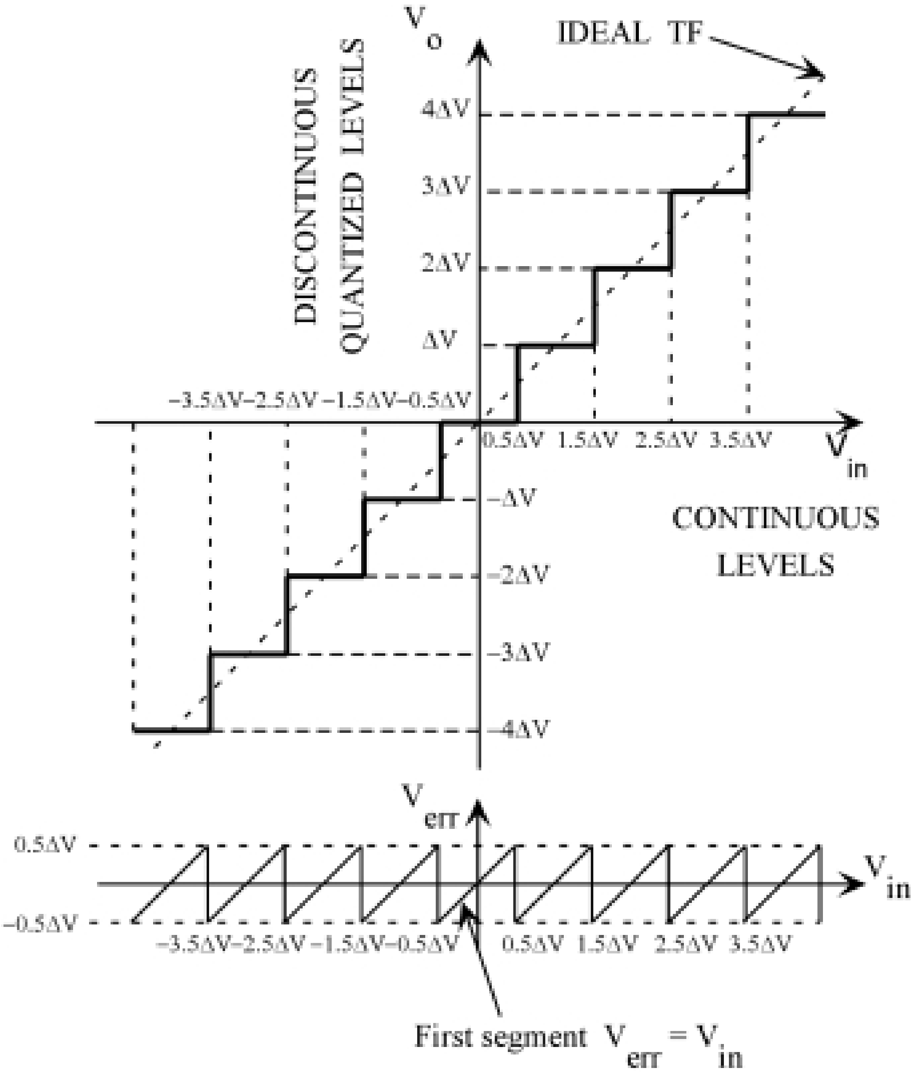
ANALOG TO DIGITAL CONVERTERS – SUPPLEMENTARY NOTES

QUANTIZATION ERROR IN A to D CONVERTERS



Let's calculate the rms error for the first segment of the error function which will be the same for all segments. ΔV is the quantizing step of the converter.

$$V_{err}(rms) = \sqrt{\frac{1}{\Delta V}\int_{-0.5\Delta V}^{+0.5\Delta V} V_{err}^{2}\, dV_{in}} = \sqrt{\frac{1}{\Delta V}\int_{-0.5\Delta V}^{+0.5\Delta V} V_{in}^{2}\, dV_{in}} = \sqrt{\frac{1}{\Delta V}\left[\frac{V_{in}^{3}}{3}\right]_{-0.5\Delta V}^{+0.5\Delta V}} = \sqrt{\frac{1}{3\Delta V}\left[\frac{\Delta V^{3}}{8}+\frac{\Delta V^{3}}{8}\right]}$$

The

$$V_{err}(rms) = \sqrt{\frac{1}{3\Delta V}\left[\frac{\Delta V^{3}}{4}\right]} = \frac{\Delta V}{\sqrt{12}}$$

quantizing error is also referred to as quantizing noise in some literature.

Now if we convert a **<u>full scale sine wave</u>**, its rms voltage is $2^N\Delta V/(2\sqrt{2})$ and the signal to quantizing noise ratio is: $\frac{S}{N_q} = \frac{2^N \Delta V \times \sqrt{12}}{2\sqrt{2}\Delta V} = 2^N \times \sqrt{1.5}$

In dB we have: $\frac{S}{N_q}(dB) = (6.02 \times N) + 1.76$ where N is # of bits of ADC

The above formula applies only to a full scale signal. As the signal level goes down, the S/N_q ratio will also go down : $\frac{S}{N_q} = fraction\ \ of\ \ FS \times 2^N \times \sqrt{1.5}$

Now in actual ADC's elecronic noise will be introduced by the internal components of the ADC and also from the input signal. Noise reduces the resolution of an ADC but distortion cuased by non-linearity of the ADC also reduces the resolution which is referred to as the effective number of bits (ENOB) . ENOB is a real measure of the ADC resolution that includes noise and distortion effects of the ADC and it is derive from the above formula:

$$ENOB = N(bits) = \frac{\frac{S}{Noise + Dist} - 1.76}{6.02} = \frac{SINAD - 1.76}{6.02}$$

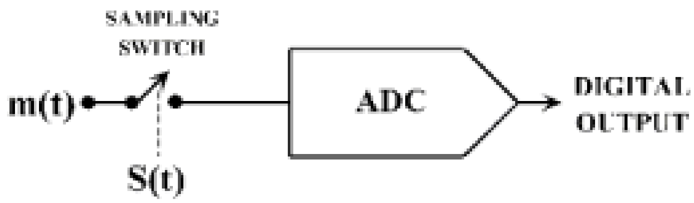
where SINAD stands for **SI**gnal to **N**oise **A**nd **D**istortion ratio.

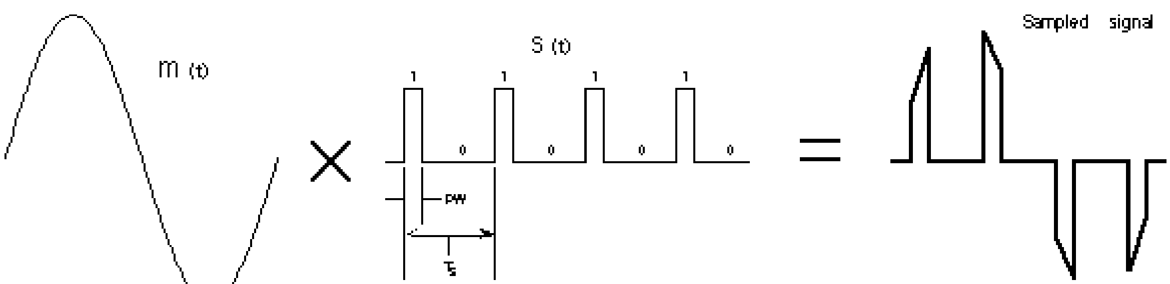
The main types of distortion encountered in ADC's comprise harmonic distortion (THD), intermodulation distortion (IMD) and aliasing errors which are caused by the sampling process. The aliasing distortion level depends mostly on the order of the filters used to attenuate the out of band frequency components which are floded back into the bandwidth of the signal by the sampling process.

Read the application notes posted on the website for more details. You can also obtain more information in standard telecom textbooks in the PAM or PCM sections.

SAMPLING THEOREM

Sampling ADC's sample the input signal at regular intervals (T_S) and encode each sample with a specific binary value. We are going to use the waveforms shown below in order to derive the frequency spectrum of a naturally sampled signal where the tops of the samples are allowed to follow m(t) during the sampling aperture (PW). Although the resulting waveform is different from that of a sampled and hold ADC (top of samples is flat), the general results are the same.





$$m_{(t)} = V_P \cos(\omega_m t) \qquad S_{(t)} = C_o + \sum_{n=1}^{\infty} C_n \cos(n\omega_s t + \phi_n)$$

$$\text{where } C_o = \frac{PW}{T_S} \quad \text{and } C_n = 2\frac{PW}{T_S} \times \frac{\sin(n\pi PW/T_S)}{(n\pi PW/T_S)}$$

$$m_{(t)} \times S_{(t)} = V_P \cos(\omega_m t) \times \left[C_o + \sum_{n=1}^{\infty} C_n \cos(n\omega_s t + \phi_n) \right]$$

$$m_{(t)} \times S_{(t)} = C_o V_P \cos(\omega_m t) + \sum_{n=1}^{\infty} \left[V_P C_n \cos(\omega_m t) \cos(n\omega_s t + \phi_n) \right]$$

$$m_{(t)} \times S_{(t)} = C_o V_P \cos(\omega_m t) + \sum_{n=1}^{\infty} \left[\frac{V_P C_n}{2} \cos((n\omega_s - \omega_m)t + \phi_n) + \frac{V_P C_n}{2} \cos((n\omega_s + \omega_m)t + \phi_n) \right]$$

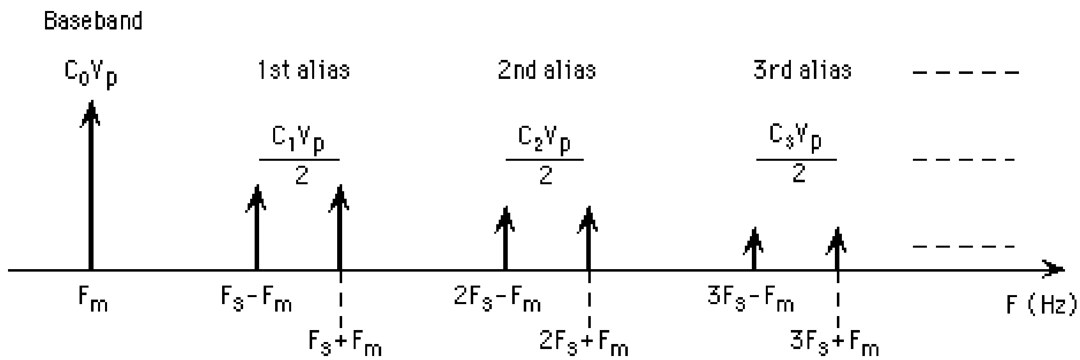
$m_{(t)} \times S_{(t)} =$

$C_o V_P COS(\omega_m t)$ baseband signal

$+\left[\frac{V_P C_1}{2} COS((\omega_s - \omega_m)t + \phi_n) + \frac{V_P C_1}{2} COS((\omega_s + \omega_m)t + \phi_n) \right]$ first alias

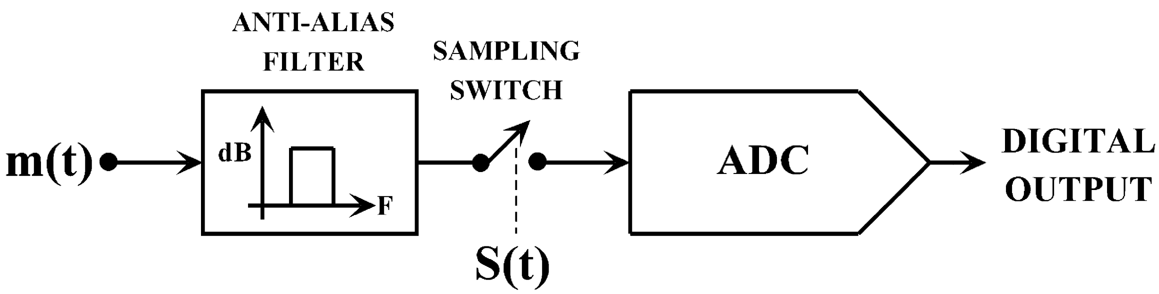
$+\left[\frac{V_P C_2}{2} COS((2\omega_s - \omega_m)t + \phi_n) + \frac{V_P C_2}{2} COS((2\omega_s + \omega_m)t + \phi_n) \right]$ second alias

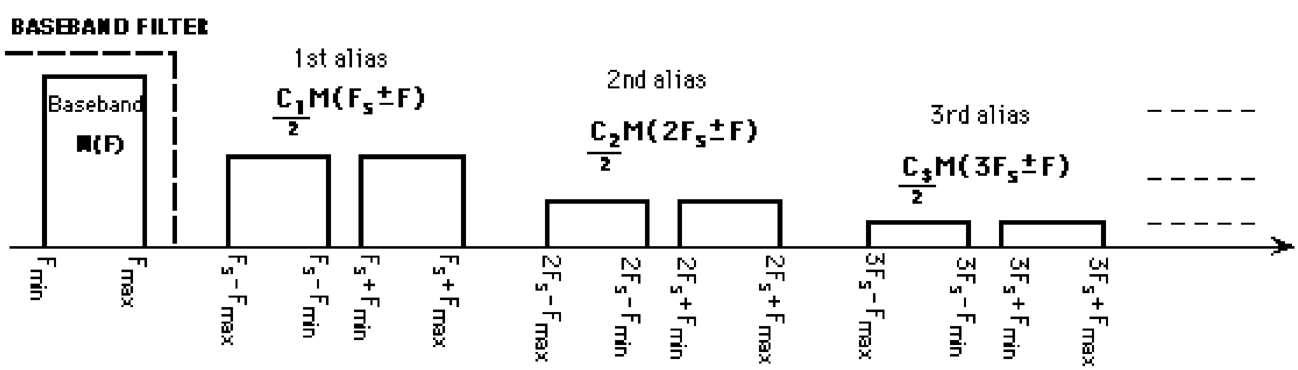
$+\left[\frac{V_P C_3}{2} COS((3\omega_s - \omega_m)t + \phi_n) + \frac{V_P C_3}{2} COS((3\omega_s + \omega_m)t + \phi_n) \right] + \bullet\bullet\bullet$ third alias



Frequency spectrum of sampled sinewave m(t) x S(t)

In practice, the baseband signal m(t) has to be filtered in order to "band-limit" it, that is to restrict the frequency range from F_{min} to F_{max}. The filter used to band-limit the signal is referred to as an anti-alias filter as shown below. The above spectrum transforms into frequency bands as shown below.



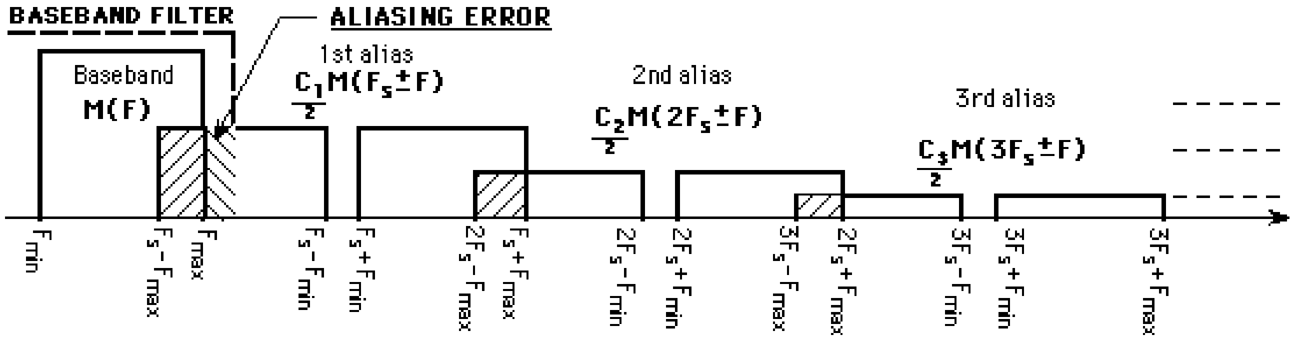


Frequency spectrum of sampled signal m(t) x S(t) where m(t) is band-limited

One can see that recovering the original baseband signal from the sampled signal is accomplished simply by using a low-pass filter that gets rid of all the aliases in the spectrum. This is only feasable if there is a gap between the baseband and the first alias, that is if $F_S - F_{max} > F_{max}$ or

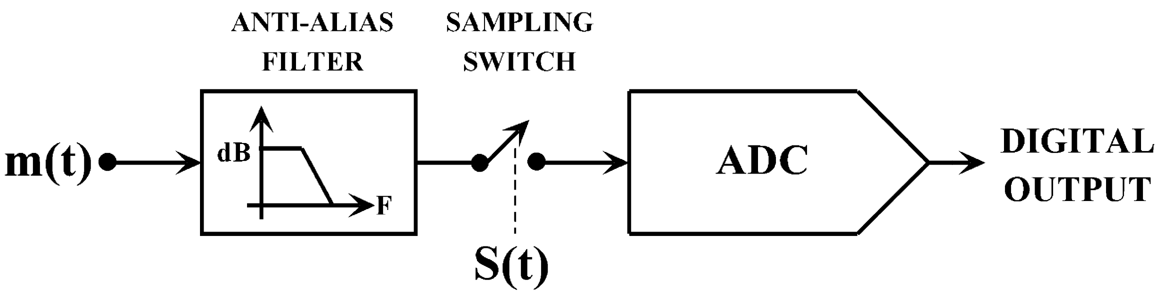
$F_S > 2F_{max}$ Nyquist sampling rate .

The above inequality states that an analog signal band-limited to F_{max} has to be sampled at a rate greater than 2 F_{max} in order to be recoverable without aliasing errors. The larger F_S is past 2 F_{max} the easier it is to filter the aliases by using a lower order filter. On the other hand, if F_S is not high enough, the first alias will overlap with the baseband and will make it impossible to recover the baseband signal without introducing aliasing errors.



Aliasing caused by low samplig rate (F_s)

ALIASING ERRORS WITH NON-IDEAL FILTERS

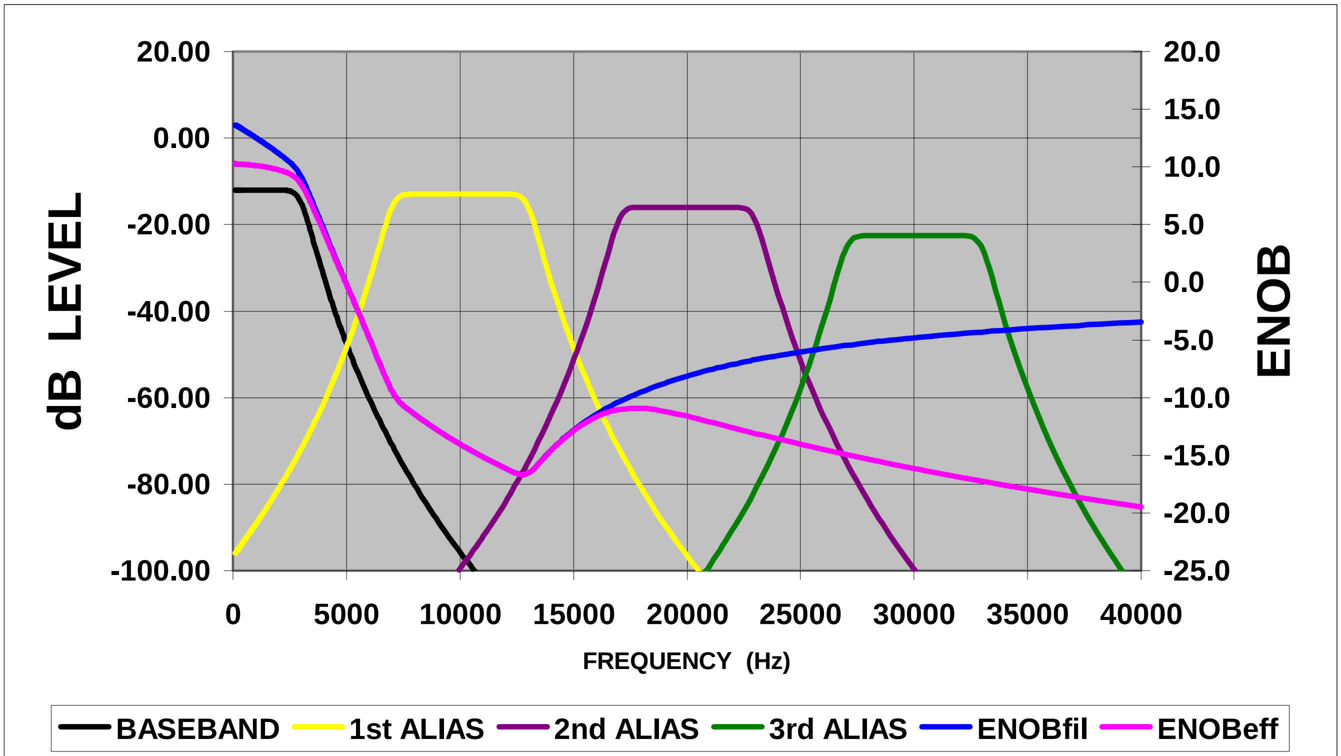


In practice the analog signal to be sampled will be band-limited with a real filter which attenuates signals outside its passband but does not eliminate them completely. Those attenuated outside frequency components will cause aliasing errors if they fold back into the passband of the filter and can ultimately decrease the ENOB of an A to D converter.

The graph below shows a sampled data system that uses a Butterworth filter and natural sampling where no sample and hold device is used – which is very rarely the case – and an 8th order filter (-160 dB/dec rolloff) with a cutoff frequency of 3 kHz. A sampling frequency of 10 kHz is assumed with a sample pulsewidth of 25 µs that causes 12 dB of attenuation (20 LOG(PW/T_S) of the sampled signal in the baseband..

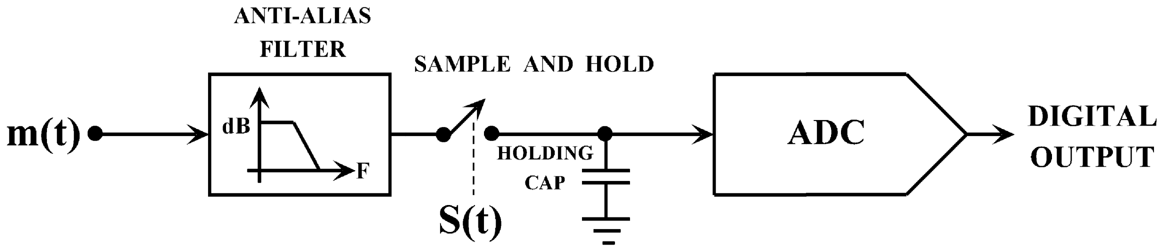
The $ENOB_{fil}$ is computed with the first alias only and indicates how much resolution the ADC loses because aliasing errors. $ENOB_{eff}$ is the total ENOB that accounts for the first alias, the ADC quantizing error and total noise referred to the ADC O/P.

As you can see below, the Butterworth filter low-pass response (in black) is aliased at nF_S-F and nF_S+F which produces mirror images of the low-pass response at harmonics of F_S.

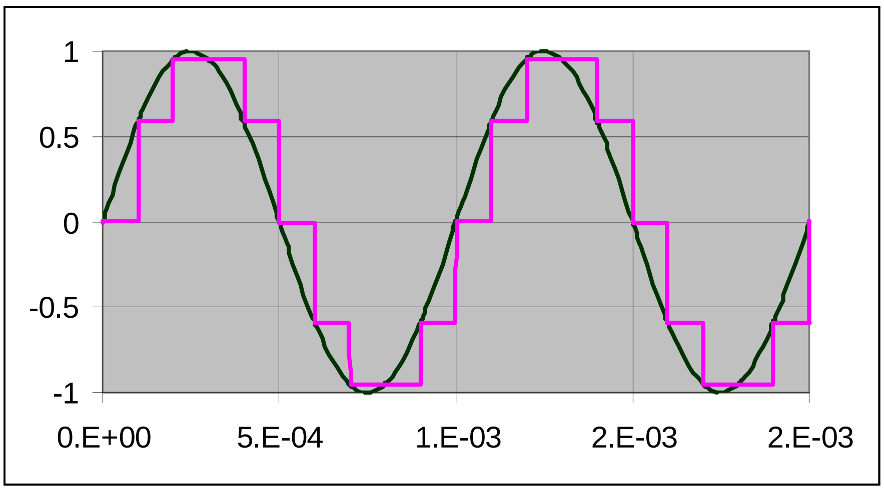


BUTTERWORTH ANTI-ALIASING LOW-PASS FILTER			SAMPLING SPEC'S			ADC SPEC'S	
			SAMPLING FREQUENCY	SAMPLE HOLD	NATURAL SAMPLING	NOISE LEVEL	ADC # BITS
Fc	ORDER	GAIN (V/V)	Fs	YES/NO	PW	dBrel (FS)	N
3000	8	1	10000	NO	2.50E-05	-80	12

SAMPLE AND HOLD ADC



In general, a sample and hold device is used just before the ADC to hold the ADC input voltage constant while an A to D conversion takes place. The S/H device is normally incorporated in the ADC chip especially in high-speed ADC's where optimum performance and full resolution of the ADC is achieved only if the S/H device is properly designed. If a S/H device is not designed properly, the ADC resolution will be impaired and a number of low significant bits will be lost.

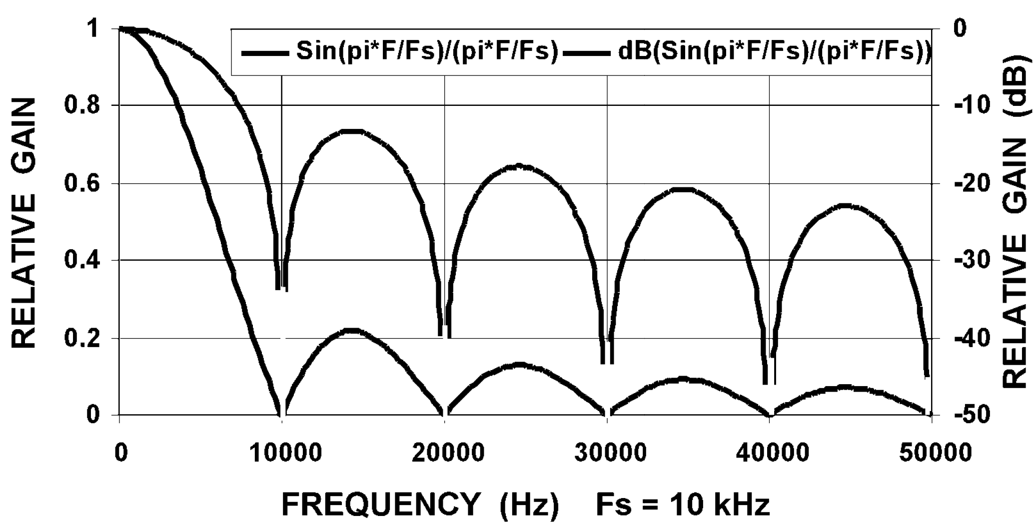


1 kHz sinewave sampled at 10 kHz or 10 ksa/s

The sample and hold process will produce aliasing errors as previously seen with natural sampling.
The S/H will introduce a second effect: it will distort the entire frequency spectrum by a factor

$$DIST_{S/H} = \frac{SIN(\pi\ F/F_S)}{(\pi\ F/F_S)}$$

which will alter the spectrum of the sampled signal.

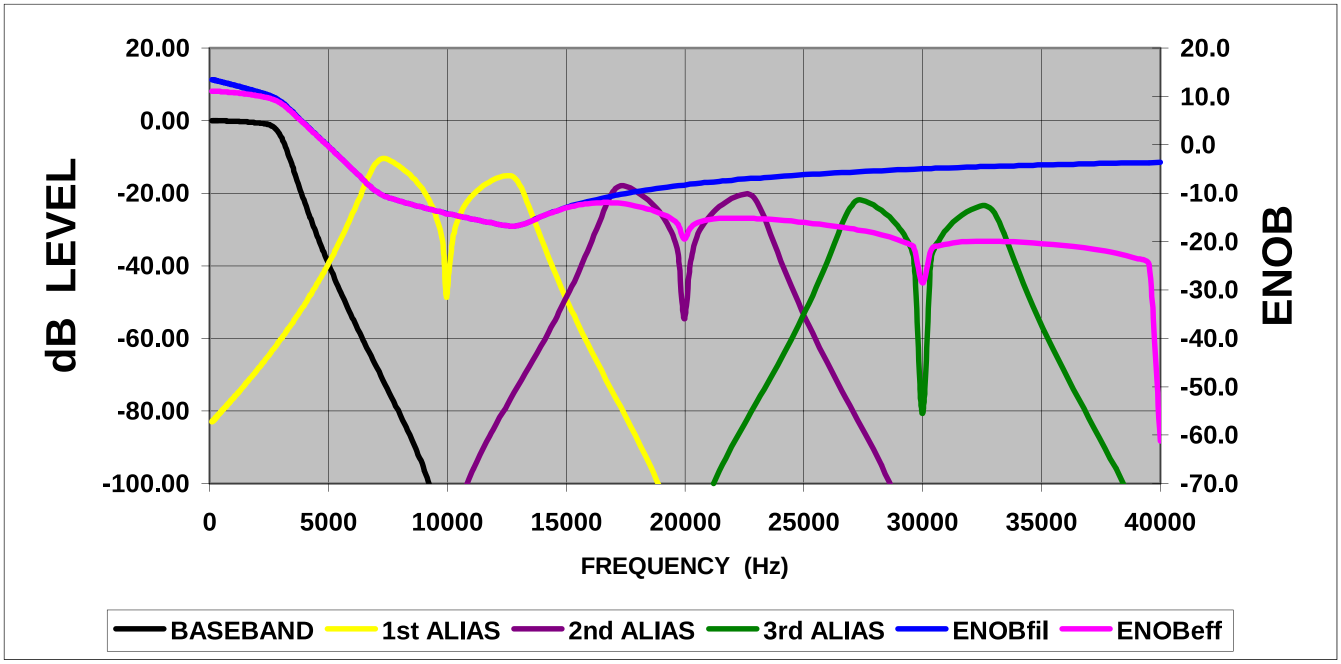


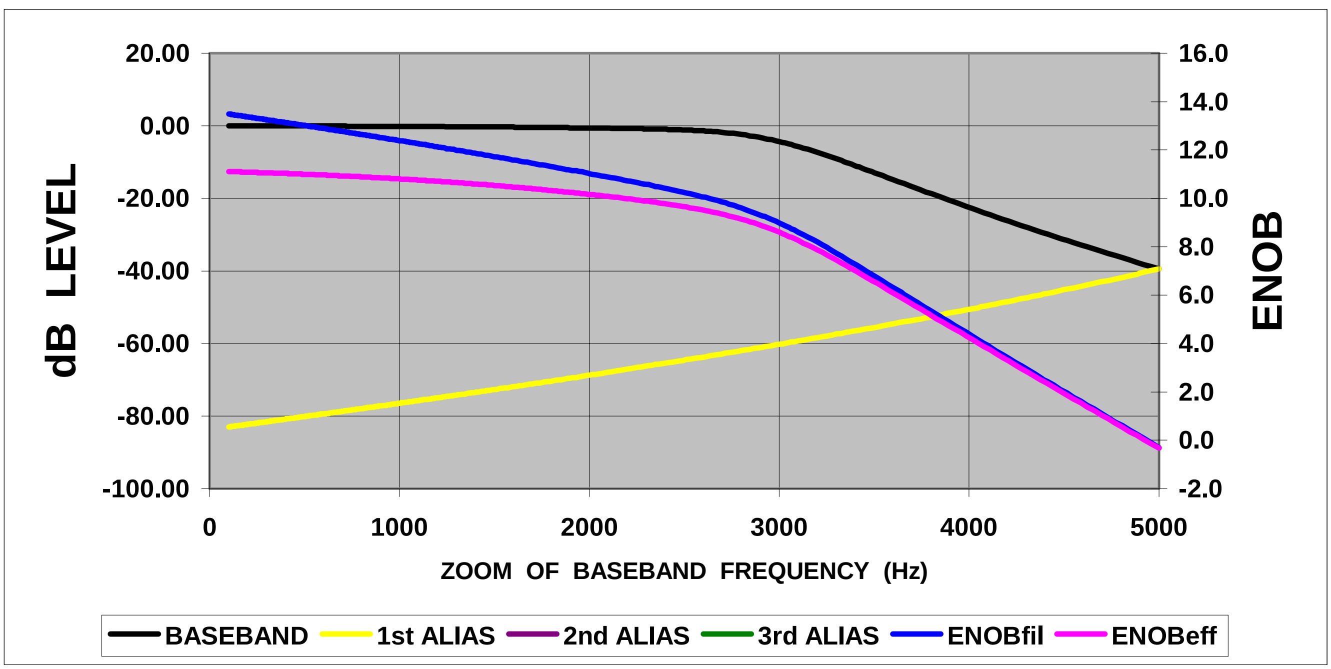
One notable effect here is that the LF components of the baseband are not attenuated as was the case with natural sampling – i.e. PW/T_S attenuation of baseband for natural sampling.

One can show that the resulting frequency components of the sample and hold O/P will be as follows:

$$m_{(t)} \times S_{H\,(t)} = \left[V_P H(j\omega) COS(\omega t) + \sum_{n=1}^{\infty} \left[\begin{matrix} V_P H\left(jn\omega_S - j\omega\right) COS\left((n\omega_s - j\omega\ )t + \phi_n\right) \\ + V_P H\left(jn\omega_S + j\omega\right) COS\left((n\omega_s + j\omega\ )t + \phi_n\right) \end{matrix} \right] \right] \times \left[\frac{SIN(\pi\ \omega / \omega_S)}{(\pi\ \omega / \omega_S)} \right]$$

where H(jω) is the anti-aliasing filter response and H(jnω$_s$± jω) is the aliased filter response mirrored about the harmonics of the sampling frequency nF$_s$.





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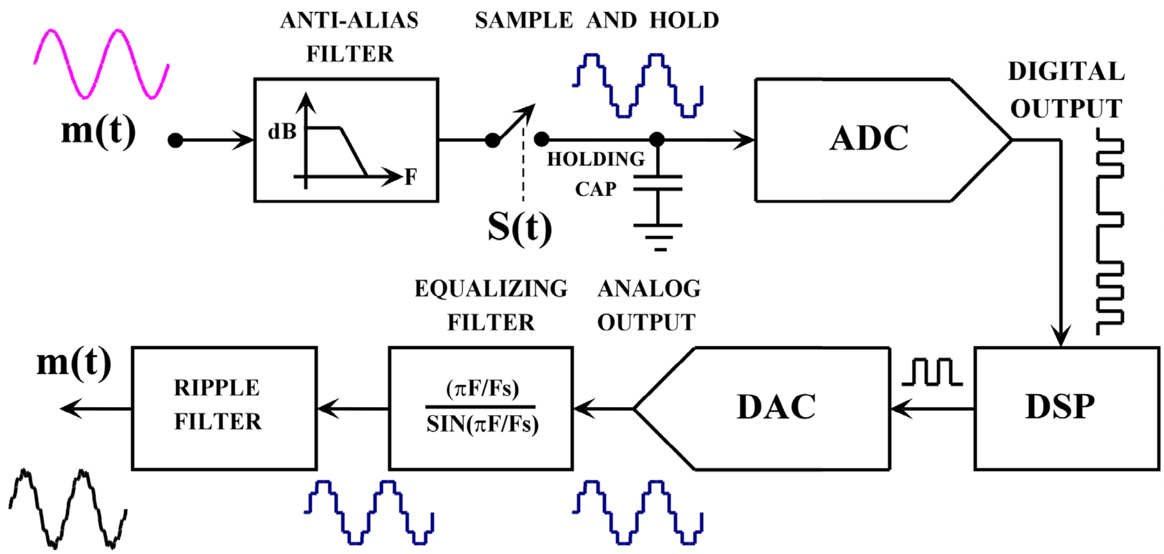
			SAMPLING FREQUENCY	SAMPLE HOLD	NATURAL SAMPLING	NOISE LEVEL	ADC # BITS
Fc	ORDER	GAIN (V/V)	Fs	YES/NO	PW	dBrel (FS)	N
3000	8	1	10000	YES	2.50E-05	-80	12

The distortion introduced by the S/H process in the baseband is not very large if $F_s >> F_{LP}$ of the low-pass anti-alias filter. If distortion is a concern, one can use an equalizing filter after the DAC when the D-to-A conversion takes place. This equalizing filter would have a TF such as

$DIST_{EQUAL} = \frac{(\pi F / F_S)}{SIN(\pi F / F_S)}$ which is the inverse of the distortion produced by the S/H. The TF needs to be $DIST_{EQUAL} = \frac{(\pi F / F_S)}{SIN(\pi F / F_S)}$ only over a limited range of frequency – baseband range 0 to F_{LP}. An additional filter is also used to filter down the steps at the DAC O/P to smooth out the signal.



For more information on DACs and ADCs and sampled data systems in general consult Analog Devices Inc. website http://www.analog.com where they have application notes – read AN-282 - and an on-line calculator for anti-aliasing filters.

IMD

Definition: intermodulation distortion. The ratio, in dB, of the total **RMS** signal level of harmonic sum and difference distortion products, to the overall RMS signal level.

The test signal is two **sine waves** added together according to the following standards:

- **SMPTE**: A 60 Hz sine wave and a 7 kHz sine wave added in a 4:1 amplitude ratio
- **DIN**: A 250 Hz sine wave and an 8 kHz sine wave added in a 4:1 amplitude ratio
- **CCIF**: A 14 kHz sine wave and a 15 kHz sine wave added in a 1:1 amplitude ratio
-

IMD reveals nonlinearities under **AC** input signal conditions as opposed to relative accuracy, which reveals nonlinearities under **DC** input signal conditions.

intermodulation distortion (SMPTE)

Definition: The ratio, in dB, of the total **RMS** signal level of harmonic sum and difference distortion products, to the overall RMS signal level. A 60 Hz **sine wave** and a 7 kHz sine wave added in a 4:1 amplitude ratio. **Units:** % or V or dB