BUTTERWORTH, BESSEL AND CHEBYCHEV CHARACTERISTICS

No.1 Draw the ideal low-pass filter gain, phase and delay characteristics and explain them. What are the advantages of an ideal filter?

No.2 Explain what amplitude distortion and group delay distortion are?

No.3 Low-pass filter : Butterworth, F_C = 2 kHz, n=4, pass-band gain of 20 dB (pages B22-B23, TF Section)

A) Determine the filter gain, phase and group delay at F = 4 kHz.
B) Determine the filter gain, phase and group delay at F = 2,8 kHz.
C) Determine the LF group delay and pass-band EDD.

No.4 High-pass filter : Bessel, F_C = 5 kHz, n=6, pass-band gain of 14 dB (pages B24-25)

A) Determine the filter gain, phase and group delay at F = 3125 Hz.
B) Determine the filter gain, phase and group delay at F = 2,5 kHz.
C) Determine the HF group delay and pass-band EDD.

No.5 Band-pass filter : Chebychev, 1 dB ripple, F_{C1} = 200 Hz, F_{C2} = 300 Hz, n=6, pass-band gain of 10 dB.

Chebychev band-pass: see pages B30 and B31 for characteristic low-pass curves and transform band-pass data to low-pass equivalent, that is $N_{LP} = N_{BP}/2 = 6/2 = 3$ and $\left(\frac{F_2 - F_1}{F_{C2} - F_{C1}}\right)_{BP} = \left(\frac{F}{F_C}\right)_{LP}$

where F_1 and F_2 are two frequencies at the same attenuation or gain level such that $F_{CEN} = \sqrt{F_{C1} \times F_{C2}} = \sqrt{F_1 \times F_2}$ in the band-pass response. F_{CEN} is the geometric center of the response.

A) Determine the filter gain, phase and group delay at F = 100 Hz.
B) Determine the filter gain, phase and group delay at F = 330 Hz.
C) Determine the mid-band group delay and pass-band EDD.

No.6 Low-pass filter: Bessel, n=4, F_C = 1 kHz, A_V = 20 dB, step input (see page A39 for details)

A) What is the maximum overshoot O/P voltage percentage wise? Show what it represents on the output waveform.
B) What is the 1% settling time to a step input? Show what it represents on the output waveform.
C) What is the approximate 0-100% rise time to a step input according to the graphic response?

No.7 Repeat no.6 with n=8.

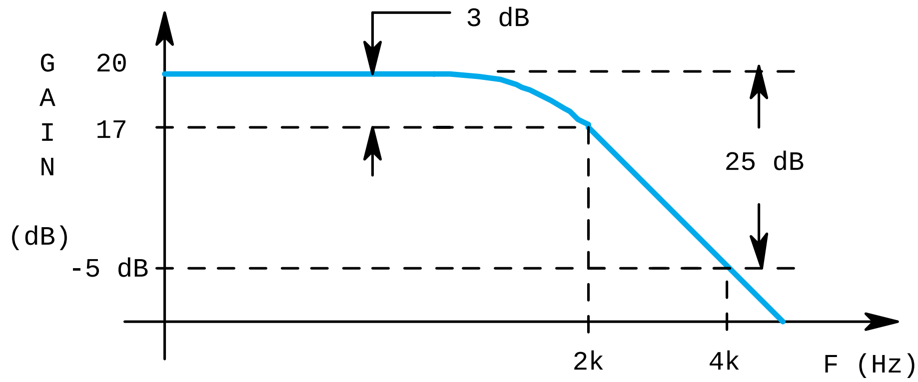
No.8 If the filter given in no.6 is implemented in the Bessel, Butterworth and Chebychev configuration, which configuration will yield the fastest settling time and the least overshoot? Explain. Refer to pages B39 and B40 of TF section.

SOLUTIONS

No.1 See course notes.

No.2 See course notes.

No.3 Butterworth low-pass, see pages B22 and B23 for characteristic curves.



A) F = 4 kHz, F/F_C = 4k/2k = 2,0 we read ATT ≈ 25 dB

Gain at 4 kHz phase ≈ -280' delay $\omega_C\ t_G \approx 0{,}8$

A_V = 20 - 25 = -5 dB $t_G = 0.8/(2\pi \times 2000)$= 63,6 μs

B) F = 2,8 kHz, F/F_C = 2,8k/2k = 1,4 we read ATT ≈ 12 dB

Gain at 2,8 kHz phase ≈ -230' delay $\omega_C\ t_G \approx 1{,}8$

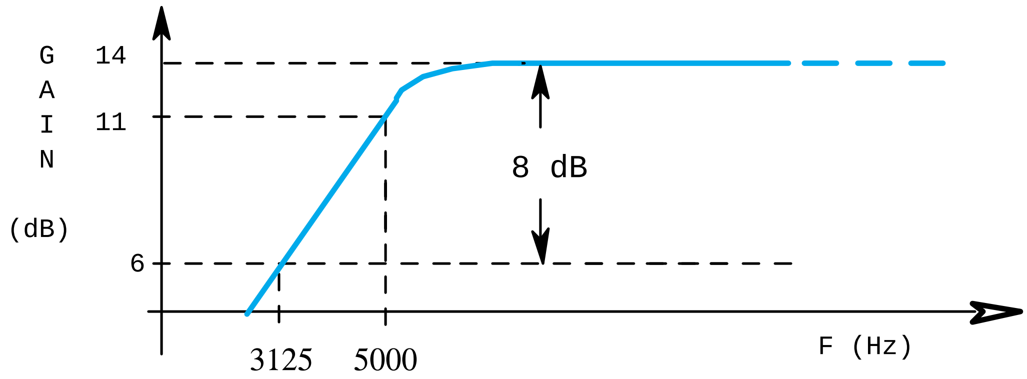
A_V = 20 - 12 = +8 dB $t_G = 1.8/(2\pi \times 2000)$= 143,2 μs

C) Low-frequency delay $\omega_C\ t_G \approx 2.75$ $t_G = 2{,}75/(2\pi \times 2000)$= 218,8 μs

Pass-band EDD is $t_{G_{\max}} - t_{G_{\min}}$ for range of F/F_C = 0 to 1,0

$$Pass-band\ EDD = t_{G\max} - t_{G\min} = \frac{\omega_C t_{G_{\max}} - \omega_C t_{G_{\min}}}{\omega_C} = \frac{4 - 2{,}75}{2\pi \times 2000} = 99{,}5\ \mu s$$

No.4 Bessel high-pass: see pages B24 and B25 for characteristic low-pass curves and transform high-pass data to low-pass equivalent, that is $(F/F_C)_{LP} = (F_C/F)_{HP}$



A) F = 5 kHz, $(F/F_C)_{LP} = (F_C/F)_{HP}$ = 5k/3125 = 1,6 we read ATT ≈ 8 dB

Gain at 3125 Hz — phase ≈ -245' — delay $\omega_C\, t_G \approx 2{,}55$

A_V = 14 - 8 = +6 dB — $t_G = 2{,}55/(2\pi \times 5000) = 81{,}2$ μs

B) F = 2,5 kHz, $(F/F_C)_{LP} = (F_C/F)_{HP}$ = 5k/2,5k = 2,0 we read ATT ≈ 13,5 dB

Gain at 2,5 kHz — phase ≈ -295' — delay $\omega_C\, t_G \approx 2{,}15$

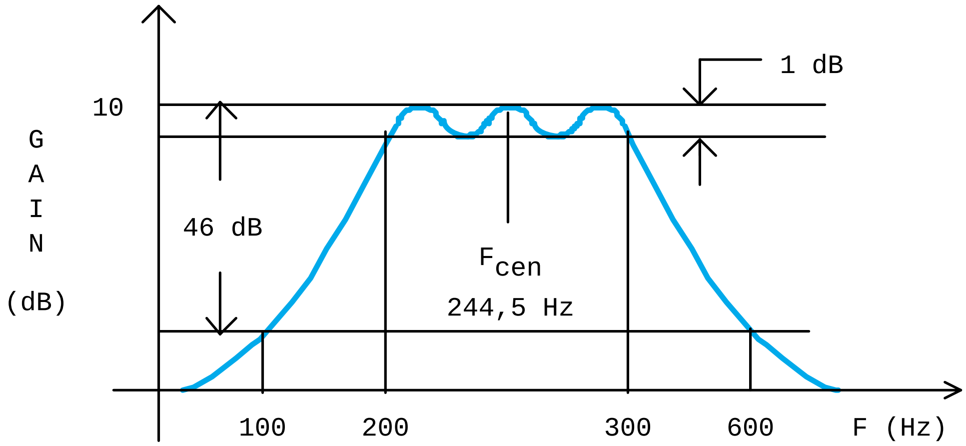
A_V = 14 - 13,5 = +0,5 dB $t_G = 2.15/(2\pi \times 5000) = 68{,}44$ μs

C) High-frequency delay $\omega_C\, t_G \approx 2{,}75$ — $t_G = 2{,}75/(2\pi \times 2000) = 218{,}8$ μs

Pass-band EDD is $t_{G_{\max}} - t_{G_{\min}}$ for range of $(F/F_C)_{LP}$ = 0 to 1, 0 or $(F_C/F)_{HP}$ = 1,00 to ∞

$$Pass-band\ EDD = t_{G_{\max}} - t_{G_{\min}} = \frac{\omega_C t_{G_{\max}} - \omega_C t_{G_{\min}}}{\omega_C} = \frac{2,7-2,7}{2\pi \times 5000} = 0\ s$$

No.5 Band-pass filter : Chebychev, 1 dB ripple, F_{C1} = 200 Hz, F_{C2} = 300 Hz, n=6, pass-band gain of 10 dB.



A) F_1 = 100 Hz

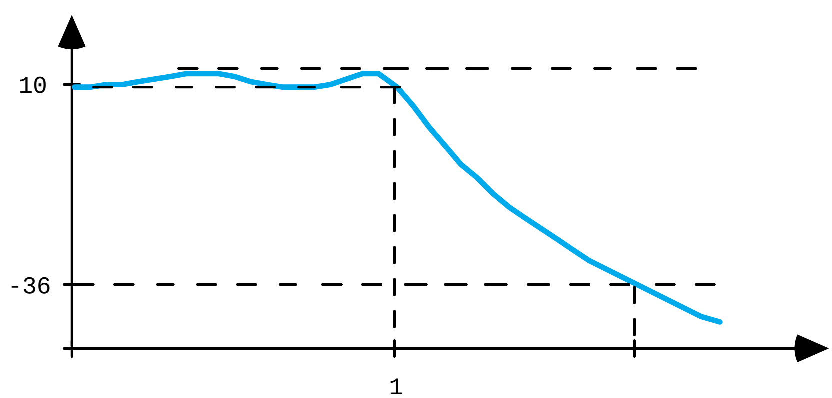
$$\left(\frac{F_2 - F_1}{F_{C2} - F_{C1}}\right)_{BP} = \left(\frac{(F_{CEN} - F_{CEN})\ to\ (F_{C2} - F_{C1})}{100}\right)_{BP} \Rightarrow \left(\frac{F}{F_C}\right)_{LP} = 0\ to\ 1$$

$$F_{CEN} = \sqrt{F_{C1} \times F_{C2}} = \sqrt{F_1 \times F_2} = \sqrt{200 \times 300} = 244{,}5\ Hz$$

$$F_1 = 100 \quad \Rightarrow \quad F_2 = F_{cen}^2 / F_1 = 244{,}5^2/100 = 600\ Hz$$

$$\left(\frac{F_2 - F_1}{F_{C2} - F_{C1}}\right)_{BP} = \left(\frac{600 - 100}{300 - 2000}\right)_{BP} = 5 \Rightarrow \left(\frac{F}{F_C}\right)_{LP} = 5$$

LOW-PASS EQUIVALENT RESPONSE



Page B31 we have: F = 100 Hz, $(F/F_C)_{LP}$ = 5 and N_{LP} = 3, we read ATT ≈ 46 dB

Gain at 100 Hz, $(F/F_C)_{LP}$ = 5
A_V = 10-46 = -36 dB

delay $(\omega_{C2}-\omega_{C1})\ t_G \approx 0$
$t_G = 0/(2\pi \times (300\text{-}200)) = 0$ s

B)

$$F_1 = 330 \quad \Rightarrow \quad F_2 = F_{cen}^2 / F_1 = 244,5^2 / 330 = 181,81\ Hz \quad \left(\frac{F_2 - F_1}{F_{C2} - F_{C1}}\right)_{BP} = \left(\frac{330 - 181,81}{300 - 200}\right)_{BP} = 1,482$$

Page B30 and B31, we have: F = 330 Hz, $(F/F_C)_{LP}$ = 1,482 and N_{LP} = 3, we read ATT ≈ 13 dB

Gain at 330 Hz, $(F/F_C)_{LP}$ = 1,482 delay $(\omega_{C2}-\omega_{C1})\ t_G \approx 1$
A_V = 10-13 = -3 dB t_G 1/(2π x (300-200))= 1,59 ms

C) $$Pass-band\ EDD = t_{G\max} - t_{G\min} = \frac{\omega_C t_{G_{\max}} - \omega_C t_{G_{\min}}}{\omega_C} = \frac{4,8-2,5}{2\pi \times (300-200)} = 3,66\ ms$$

No.6 Low-pass filter: Bessel, n=4, F_C = 1 kHz, A_V = 20 dB, step input (see page A39 for details)

A) Max overshoot = 0,8% B) $t_{1\%}$ = 1/F_C = 1 ms C) $t_r \approx 2/(\pi F_C)$= 0,637 ms
Explanations are yours.

No.7 Low-pass filter: Bessel, n=8, F_C = 1 kHz, A_V = 20 dB, step input (see page A39 for details)

A) Max overshoot = 0,3% B) $t_{1\%}$ = 1,6/F_C=1,6 ms C) $t_r \approx 2,5/(\pi F_C)$= 0,8 ms
Explanations are yours.

No.8 Use table on page B40 of TF theory section. 4th order low-pass filters, same F_C = 1 kHz

TYPE	Butterworth	Bessel	0.5 dB Cheb	2 dB Cheb
Max overshoot	11%	**0.8%**	18%	28%
1% settling	1.7/F_C	**1/F_C**	3/F_C	4.8/F_C

Explanations are yours.